Aharonov-Bohm Effect and Scattering Theory

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The Aharonov-Bohm effect is a quantum mechanical phenomenon wherein charged particles, like electrons, are physically influenced by the existence of magnetic fields in regions that are inaccessible to the particles. This is different from the case of classical mechanics where the force is zero if the magnetic field vanishes. This effect is a fundamental issue in physics. It describes the physically important electromagnetic quantities in quantum mechanics, and its experimental verification constitues a test of the theory of quantum mechanics itself. In remarkable experiments Tonomura et al. enclosed a magnetic flux inside a toroidal magnet and they superimposed behind the magnet an electron wave packet that traveled inside the hole of the magnet with a reference electron wave packet that traveled outside the magnet. They measured the phase shift produced by the magnetic flux inside the magnet. These experiments gave a strong evidence of the physical existence of the Aharonov-Bohm effect. In this talk I study the Aharonov-Bohm effect from the point of view of time-dependent scattering theory. Under the assumption that the incoming free electron is a gaussian wave packet, I estimate the exact solution to the Schrödinger equation for all times. I provide a rigorous, quantitative, error bound for the difference between the exact solution and the approximate solution given by the Aharonov-Bohm Ansatz. The error bound is uniform in time. I also prove that on the gaussian asymptotic state, the scattering operator is given by multiplication by $e^{i\frac{q}{\hbar c}\tilde{\Phi}}$ -where q is the charge of the electron, c is the speed of light, $\hbar$ is Planck's constant, and $\tilde{\Phi}$ is the magnetic flux in a transversal section of the magnet- up to a quantitative error bound, that I provide. Using the experimental data of

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Tonomura et al., I evaluate the error bound and I provide rigorous lower and upper bounds on the variance of the gaussian state in order that quantum mechanics predicts the results of the Tonomura et al. experiments, up to an error that can be very small. The error bound has a physical interpretation. The lower bound is due to Heisenberg's uncertainty principle. If the variance in configuration space is small, the variance in momentum space is big, and then, the component of the momentum transversal to the axis of the magnet is large. In consequence, the opening angle of the electron wave packet is large, and there is a large interaction with the magnet. If the variance is large, the opening angle is small, but as the electron wave packet is big we have again a large interaction with the magnet. These results show that it would be quite interesting to perform experiments for a medium size electron wave packet -smaller than the ones used in the Tonomura et al. experiments, that where much larger that the magnet- that satisfies the lower and upper bounds. One could as well take a larger magnet. In this case, the interaction of the electron wave packet with the magnet is negligible and, moreover, quantum mechanics predicts the results observed by Tonomura et al. with a very small error bound.
In the second part of my talk I go beyond the case of the Tonomura et al. experiments. I consider the situation where the magnet is a general, bounded, handle-body. In particular, a finite number of tori. I assume that in addition to the magnetic flux inside the magnet there are a magnetic field and an electric potential outside the magnet. I study incoming electron wave packets that have negligible interaction with the handle-body in the high-velocity limit. I present high-velocity estimates, with error bounds, for the scattering operator of the Schrödinger equation in three dimensions. I give a method for the reconstruction, modulo 2π, of the magnetic flux across the transversal section of each connected component of the handle-body. I also give a method for the reconstruction of the electric potential and the magnetic field outside the handle-body. The results of this talk are joint work with Miguel Ballesteros, and where proven using the method introduced in [2, 8].

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